Published by Institute of Physics Publishing for SISSA/ISAS

Received: *May 1, 2004*
Revised: *September 20, 2004*
Accepted: *September 21, 2004*

Mass singularity and confining property in QED$_3$

Yuichi Hoshino

Kushiro National College of Technology
Otanoshike nishi 2-32-1, Kushiro, Hokkaodo, 084-0096, Japan
E-mail: hoshino@marimo.kushiro-ct.ac.jp

Abstract: We discuss the properties of the position space fermion propagator in three dimensional QED which has been found previously based on Ward-Takahashi–identity for soft-photon emission vertex and spectral representation in quenched approximation.There is a new type of mass singularity which governs the long distance behaviour.It leads the propagator vanish at large distance.This term corresponds to dynamical mass in position space and expressed as superposition of the propagator with different mass in momentum space.Our model shows confining property and dynamical mass generation for arbitrary coupling constant.Since we used dispersion relation in deriving spectral function there is a physical mass which sets a mass scale. Low energy behaviour of the proagator is modified to decrease by position dependent mass.In the limit of zero infrared cut-off the propagator vanishes with new kind of infrared behaviour.

Keywords: Field Theories in Lower Dimensions, Nonperturbative Effects, Spontaneous Symmetry Breaking.

Contents

1. Introduction

Infrared behaviour of the propagator in the presence of massless particle (i.e. photon and graviton) had been discussed assuming spectral representation of the propagator and the use of Ward-Takahashi-identity [1]. It is pointed out that the spectral representation is model independent and the asymptotic form of the scalar propagator in QED and Gravity theory were determined [1]. It has long been thought that the three dimensional QED is a confining theory since its infrared divergences is severe [5]. However it has not been known what we can observe as a confinement. One of these examples is a vanishment of the propagator in the infrared region. In the previous paper we find that the scalar and spinor propagator have a new type of mass singularity in the same approach in ref. [1] to QED_3 [4]. There are two kinds of gauge invariant mass singularities in the evaluation of the $O(e^2)$ spectral function by LSZ reduction formula. After exponentiation of $O(e^2)$ spectral function we get the full propagator in position space which is a product of free propagator with physical mass and quantum correction. In our model quantum corrections are Coulomb energy and position dependent mass which are both logarithmically divergent at long distance. Here confinement means that the propagator damps faster than the free propagator with physical mass for arbitrary coupling constant. First we study the structure of the propagator with finite infrared cut-off μ. It is assumed that the three dimensional analysis leads the leading order of high temperature expansion results in four dimension [5]. Using Laplace transformation of the position dependent mass numerically,

JHEP09(2004)048

we have the integral reresentation of the propagator in momentum space as in dispersion theory. As a result we find in momentum space that the propagator vanishes in the limit of zero infrared cut-off $\mu^2 = m^2 - p^2$ near $p^2 = m^2$. In section 2 Bloch-Nordsieck approximation in three dimension is reviewed. In section 3 mass singularity in four and three dimension is compared and numerical analysis in momentum space is given. Section 4 is for non-perturbative effects as renormalization constant,bare mass which are defined in the high-energy limit,and the vacuum expectation value of the composite operator $\langle\overline{\psi}\psi\rangle$. Renormalization constant Z_2^{-1} vanishes for arbitrary coupling constant.There seems to be a critical coupling constant which separates the phases of $\langle\overline{\psi}\psi\rangle$.

2. Ward-Takahashi–identity and the spectral function

2.1 Position space propagator

First we consider about the charged particles which emitt and absorb massless photons. Usually this process was described by spectral function;transition probability of particle into particle and photon state. This method is model independent and helpful in any dimension and leads to the so called Bloch-Nordsieck approximation. Multi-photon emitted from external line is introduced by ladder type diagrams which satisfy Ward-Takahashi–identity. Let us begin by dispersion theoretic description of the propagator [2]

$$\begin{aligned} S_F(p) &= \int d^3x \exp(-ip\cdot x)\left\langle\Omega|T\psi(x)\overline{\psi}(0)|\Omega\right\rangle \\ &= \int d\omega^2 \frac{\gamma\cdot p\rho_1(\omega^2)+\rho_2(\omega^2)}{p^2-\omega^2+i\epsilon}, \end{aligned} \tag{2.1}$$

$$\begin{aligned} \frac{1}{\pi}\,\mathrm{Im}\,S(p) &= \int d\omega^2\delta(p^2-\omega^2)[\gamma\cdot p\rho_1(\omega^2)+\rho_2(\omega^2)] \\ &= \gamma\cdot p\rho_1(p)+\rho_2(p)\,, \end{aligned} \tag{2.2}$$

$$S_F(x) = \left\langle\Omega|T\psi(x)\overline{\psi}(0)|\Omega\right\rangle . \tag{2.3}$$

The field ψ is renormalized and is taken to be a spinor with mass m. Here we introduce intermediate states that contribute to the spectral function

$$\begin{aligned} \rho(p) &= \gamma\cdot p\rho_1(p)+\rho_2(p) \\ &= (2\pi)^2\sum_N \delta^3(p-p_N)\int d^3x \exp(-ip\cdot x)\left\langle\Omega|\psi(x)|N\right\rangle\left\langle N|\overline{\psi}(0)|\Omega\right\rangle . \end{aligned} \tag{2.4}$$

Total three-momentum of the state $|N\rangle$ is p^μ_N.The only intermediates N contain one spinor and an arbitrary number of photons.Setting

$$|N\rangle = |r;k_1,\ldots,k_n\rangle, \tag{2.5}$$

where r is the momentum of the spinor $r^2 = m^2$,and k_i is the momentum of ith soft photon,we have

$$\begin{aligned} \rho(p^2) &= \int \frac{md^2r}{r^0}\sum_{n=0}^{\infty}\frac{1}{n!}\times\left(\int\frac{d^3k}{(2\pi)^3}\theta(k_0)\delta(k^2\right)\sum_{\epsilon})_n\delta\left(p-r-\sum_{i=1}^{n}k_i\right)\times \\ &\times\left\langle\Omega|\psi(x)|r;k_1,\ldots,k_n\right\rangle\left\langle r;k_1,\ldots,k_n|\overline{\psi}(0)|\Omega\right\rangle . \end{aligned} \tag{2.6}$$

Here the notations

$$(f(k))_0 = 1, (f(k))_n = \prod_{i=1}^{n} f(k_i) \tag{2.7}$$

have been introduced to denote the phase space integral of each photon. Intial sum over ϵ is a sum over polarization of photon. To evaluate the contribution of soft-photons, we consider when only nth photon is soft. Here we define following matrix element

$$T_n = \langle \Omega | \psi | r; k_1, \ldots, k_n \rangle . \tag{2.8}$$

We consider T_n for $k_n^2 \neq 0$,we continue off the photon mass-shell by Lehmann-Symanzik-Zimmermann(LSZ)formula:

$$\begin{aligned}
T_n &= \epsilon_n^\mu T_{n\mu} \,, && (2.9)\\
\epsilon_n^\mu T_n^\mu &= \left\langle \Omega | T\psi a_n^{+\epsilon}(k) | r; k_1, \ldots, k_{n-1} \right\rangle && (2.10)\\
&= i \lim_{t_i \to -\infty} \int_{t_i} d^3x \exp(ik_n \cdot x) \overleftrightarrow{\partial_0} \left\langle \Omega | T\psi \epsilon^\mu A_\mu^{T,in}(x) | r; k_1, \ldots, k_{n-1} \right\rangle && (2.11)\\
&= \frac{i}{\sqrt{Z_3}} \int d^3x \exp(ik_n \cdot x)(\Box_x + \mu^2) \left\langle \Omega | T\psi \epsilon^\mu A_\mu(x) | r; k_1, \ldots k_{n-1} \right\rangle && (2.12)\\
&= -\frac{i}{\sqrt{Z_3}} \int d^3x \exp(ik_n \cdot x) \left\langle \Omega | T\psi \epsilon_\mu j^\mu | r; k_1, \ldots k_{n-1} \right\rangle , && (2.13)
\end{aligned}$$

provided

$$\begin{aligned}
(\Box_x + \mu^2) T\psi A_\mu(x) &= T\psi(\Box_x + \mu^2) A_\mu(x) = T\psi(-j_\mu(x) + (1-\lambda)\partial_\mu^x(\partial \cdot A(x))) \,,\\
\lambda \left(\Box_x + \frac{\mu^2}{\lambda} \right) (\partial \cdot A) &= 0 \,, \quad (\Box_x + \mu^2) \frac{\mu^2}{\lambda} \partial(\partial \cdot A(x)) = (\lambda - 1)\partial_\mu(\partial \cdot A(x)) \,, && (2.14)
\end{aligned}$$

where the electromagnetic current is

$$j^\mu(x) = -e\overline{\psi}(x)\gamma_\mu \psi(x), \tag{2.15}$$

and $1/\lambda$ is a gauge fixing parameter and μ is a photon mass. In deriving the spectral representation for photon propagator, the Stueckelberg approach was used [3] and the realistic condition for transverse component of incoming field is defined

$$A_\mu^{T,in}(x) = A_\mu^{in}(x) + \frac{\lambda^2}{\mu^2} \partial_\mu \partial \cdot A^{in}(x) \,, \tag{2.16}$$

and assumed

$$A_\mu(x)_{t \to -\infty} \to \sqrt{Z_3} \left[A_\mu^{T,in} - z\frac{\lambda^2}{\mu^2} \partial_\mu \partial \cdot A^{in}(x) \right] . \tag{2.17}$$

Condition for matrix element in particular is

$$\langle 0 | A_\mu(x) | 1 \rangle = \sqrt{Z_3} \left[\left\langle 0 | A_\mu^{T,in}(x) | 1 \right\rangle - z\frac{\lambda^2}{\mu^2} \partial_\mu \left\langle 0 | \partial \cdot A^{in}(x) | 1 \right\rangle \right] , \tag{2.18}$$

where z is a renormalization constant for londitudinal part [3]. An alternative way to derive the reduction formula is an imposing Lorentz condition for the physical state

$$\partial \cdot A^{(+)}|\text{phys} >= 0\,. \tag{2.19}$$

From the definition (2.9), (2.10) T_n is seen to satisfy Ward-Takahashi-identity:

$$k_{n\mu}T_n^{\mu}(r,k_1,\ldots,k_n) = eT_{n-1}(r,k_1,\ldots,k_{n-1}), r^2 = m^2\,, \tag{2.20}$$

provided by equal-time commutation relations

$$\begin{aligned}\partial_\mu^x T(\psi j_\mu(x)) &= -e\psi(x)\,,\\ \partial_\mu^x T(\overline{\psi} j_\mu(x)) &= e\overline{\psi}(x)\,.\end{aligned} \tag{2.21}$$

In the Bolch-Nordsieck approximation we have most singular contribution of photons which are emitted from external lines. In perturbation theory one photon matrix element is given

$$\begin{aligned}T_1 &= \left\langle in|T(\psi_{in}(x), ie\int d^3x\overline{\psi}_{in}(y)\gamma_\mu\psi_{in}(y)A^\mu_{in}(y))|r;k\ in\right\rangle\\ &= ie\int d^3y d^3z S_F(x-y)\gamma_\mu\delta^{(3)}(y-z)\exp(i(k\cdot y + r\cdot z))\epsilon^\mu(k,\lambda)U(r,s)\\ &= -ie\frac{(r+k)\cdot\gamma + m}{(r+k)^2 - m^2}\gamma_\mu\epsilon^\mu(k,\lambda)\exp(i(k+r)\cdot x)U(r,s)\,,\end{aligned} \tag{2.22}$$

where $U(r,s)$ is a four-component free particle spinor with positive energy. $U(r,s)$ satisfies the relations

$$\begin{aligned}(\gamma\cdot r - m)U(r,s) &= 0\,,\\ \sum_S U(r,s)\overline{U}(r,s) &= \frac{\gamma\cdot r + m}{2m}\,.\end{aligned} \tag{2.23}$$

In this case the Ward-Takahashi-identity follows

$$\begin{aligned}k_\mu T_1^\mu &= -ie\frac{1}{\gamma\cdot(r+k) - m}(\gamma\cdot k)U(r,s)\\ &= -ieU(r,s) = eT_0\,,\end{aligned} \tag{2.24}$$

provided lowest-order Ward-identity

$$\gamma\cdot k = (\gamma\cdot(r+k) - m) - (\gamma\cdot k - m)\,. \tag{2.25}$$

For general T_n low-energy thorem determines the structure of non-singular term in k_n by the requirement of gauge invariance of total transion amplitudes under the shift $\epsilon^\mu \to \epsilon^\mu + ck_n^\mu$.They give finite correction to the pole terms for examples in the Bremuthstralung or Compton scattering. Detailed discussions are given in ref. [2] and non-pole terms are irerevant for the single paticle singularity in four-dimension. Under the same assumption in three-dimension we have

$$T_n|_{k_n^2=0} = e\frac{\gamma\cdot\epsilon}{\gamma\cdot(r+k_n) - m}T_{n-1}\,. \tag{2.26}$$

From this relation the n-photon matrix element

$$\langle\Omega|\psi(x)|r;k_1,\ldots,k_n\rangle\left\langle r;,k_{1,}\ldots,k_n|\overline{\psi}(0)|\Omega\right\rangle \tag{2.27}$$

reduces to the products of lowest-order one photon matrix element

$$\prod_{j=1}^{n}T_1(k_j)T^{+}(k_j)\gamma_0\rightarrow\prod_{j=1}^{n}T_1(k_j)T_1^{+}(k_j)\gamma_0 \tag{2.28}$$

In this case the spectral function $\rho(p)$ in (2.6) is given by exponentiation of one-photon matrix element, which yields a infinite ladder approximation for the propagator. In this way the spectral function is given in the followings

$$\overline{\rho(x)} = \int\frac{md^2r}{(2\pi)^2r^0}\exp(ir\cdot x)\exp(F), \tag{2.29}$$

$$\begin{aligned} F &= \sum_{\text{one photon}}\langle\Omega|\psi(x)|r;k\rangle\left\langle r;k|\overline{\psi}(0)|\Omega\right\rangle \\ &= \int\frac{d^3k}{(2\pi)^2}\delta(k^2)\theta(k_0)\exp(ik\cdot x)\sum_{\lambda,S}T_1\overline{T_1}\,. \end{aligned} \tag{2.30}$$

To determine F first we take the trace of $T_1\overline{T_1}$ for simplicity in the infrared.

$$\begin{aligned} F = \int&\frac{d^3k}{(2\pi)^2}\exp(ik\cdot x)\delta(k^2)\theta(k_0)\times \\ &\times\frac{e^2}{4}\,\mathrm{tr}\left[\frac{(r+k)\cdot\gamma}{(r+k)^2-m^2}\gamma^{\mu}\frac{r\cdot\gamma+m}{2m}\gamma^{\nu}\frac{(r+k)\cdot r}{(r+k)^2-m^2}\Pi_{\mu\nu}\right]. \end{aligned} \tag{2.31}$$

Here $\Pi_{\mu\nu}$ is the polarization sum

$$\Pi_{\mu\nu}=\sum_{\lambda}\epsilon_{\mu}(k,\lambda)\epsilon_{\nu}(k,\lambda)=-g_{\mu\nu}-(d-1)\frac{k_{\mu}k_{\nu}}{k^2}\,, \tag{2.32}$$

and the free photon propagator is

$$D_0^{\mu\nu}=\frac{1}{k^2+i\epsilon}\left[g_{\mu\nu}+(d-1)\frac{k_{\mu}k_{\nu}}{k^2}\right]. \tag{2.33}$$

We get

$$F=-e^2\int\frac{d^3k}{(2\pi)^3}\exp(ik\cdot x)\theta(k_0)\left[\delta(k^2)\left(\frac{m^2}{(r\cdot k)^2}+\frac{1}{r\cdot k}\right)+(d-1)\frac{\delta(k^2)}{k^2}\right]. \tag{2.34}$$

The second term $\delta(k^2)/k^2$ equals to $-\delta'(k^2)$.Our calculation is the same with the evaluation of the imaginay part of the photon propagator. To aviod the infrared divergences which arises in the phase space integral, we must introduce small photon mass μ as an infrared cut-off. Therefore (2.34) is modified to

$$\begin{aligned} F = -e^2\int&\frac{d^3k}{(2\pi)^3}\exp(ik\cdot x)\theta(k_0)\times \\ &\times\left[\delta(k^2-\mu^2)\left(\frac{m^2}{(r\cdot k)^2}+\frac{1}{(r\cdot k)}\right)-(d-1)\frac{\partial}{\partial k^2}\delta(k^2-\mu^2)\right]. \end{aligned} \tag{2.35}$$

Here we assume $\rho_1(\omega^2)/m = \rho_2(\omega^2) = \rho$ which is valid in the infrared (i.e. $\gamma \cdot r = m$).In general case there are two kinds of spectral function which is given in the appendix. It is helpful to use function $D_+(x)$ to determine F

$$\begin{aligned} D_+(x) &= \frac{1}{(2\pi)^2 i} \int \exp(ik \cdot x) d^3k \theta(k^0) \delta(k^2 - \mu^2) \\ &= \frac{1}{(2\pi)^2 i} \int_0^\infty J_0(k\,|x|) \frac{\pi k dk}{2\sqrt{k^2+\mu^2}} = \frac{\exp(-\mu\,|x|)}{8\pi i\,|x|}, \\ |x| &= \sqrt{-x^2}\,. \end{aligned} \tag{2.36}$$

If we use parameter trick

$$\lim_{\epsilon \to 0} \int_0^\infty d\alpha \exp(i(k+i\epsilon) \cdot (x + \alpha r)) = \frac{i \exp(ik \cdot x)}{k \cdot r}, \tag{2.37}$$

$$\lim_{\epsilon \to 0} \int_0^\infty \alpha d\alpha \exp(i(k+i\epsilon) \cdot (x + \alpha r)) = -\frac{\exp(ik \cdot x)}{(k \cdot r)^2}, \tag{2.38}$$

the function F is written in the following form

$$\begin{aligned} F &= ie^2 m^2 \int_0^\infty \alpha d\alpha D_+(x + \alpha r, \mu) - e^2 \int_0^\infty d\alpha D_+(x + \alpha r, \mu) - ie^2(d-1) \frac{\partial}{\partial \mu^2} D_+(x, \mu) \\ &= \frac{e^2 m^2}{8\pi r^2} \left(-\frac{\exp(-\mu\,|x|)}{\mu} + |x| \,\mathrm{Ei}(\mu\,|x|) \right) - \frac{e^2}{8\pi\sqrt{r^2}} \mathrm{Ei}(\mu\,|x|) + (d-1) \frac{e^2}{8\pi\mu} \exp(-\mu\,|x|) \\ &= F_1 + F_2 + F_g\,, \end{aligned} \tag{2.39}$$

where the function $\mathrm{Ei}(\mu\,|x|)$ is defined

$$\mathrm{Ei}(\mu\,|x|) = \int_1^\infty \frac{\exp(-\mu\,|x|\,t)}{t} dt\,. \tag{2.40}$$

It is understood that all terms which vanishes with $\mu \to 0$ are ignored. The leading non trivial contributions to F are

$$\mathrm{Ei}(\mu\,|x|) = -\gamma - \ln(\mu\,|x|) + O(\mu\,|x|)\,, \tag{2.41}$$

$$\begin{aligned} F_1 &= \frac{e^2 m^2}{8\pi r^2} \left(-\frac{1}{\mu} + |x| \,(1 - \ln(\mu\,|x|) - \gamma) \right) + O(\mu)\,, \\ F_2 &= \frac{e^2}{8\pi\sqrt{r^2}} (\ln(\mu\,|x|) + \gamma) + O(\mu)\,, \\ F_g &= \frac{e^2}{8\pi} \left(\frac{1}{\mu} - |x| \right) (d-1) + O(\mu)\,, \end{aligned} \tag{2.42}$$

$$F = \frac{e^2}{8\pi\mu}(d-2) + \frac{\gamma e^2}{8\pi m} + \frac{e^2}{8\pi m} \ln(\mu\,|x|) - \frac{e^2}{8\pi} |x| \ln(\mu\,|x|) - \frac{e^2}{8\pi} |x| \,(d - 2 + \gamma)\,, \tag{2.43}$$

where γ is Euler's constant and we set $r^2 = m^2$.Here we used integrals for intermediate state for on-shell fermion

$$\int d^3x \exp(-ip \cdot x) \int d^3r \delta(r^2 - m^2) \exp(ir \cdot x) f(r) = f(m)\,, \tag{2.44}$$

$$\int d^3x \exp(-ip \cdot x) \int d^3r \frac{\exp(-mx)}{4\pi x} \exp(ir \cdot x) \delta(r^2 - m^2) = \frac{1}{m^2 + p^2}\,. \tag{2.45}$$

Exponentiation of F in (2.43) leads simple form

$$\exp(F) = \exp(A - B\,|x|)(\mu\,|x|)^{D-C|x|}\,, \tag{2.46}$$

where

$$A = \frac{e^2}{8\pi\mu}(d-2) + \frac{\gamma e^2}{8\pi m}\,, \quad B = \frac{e^2}{8\pi}(d-2+\gamma)\,, \quad C = \frac{e^2}{8\pi}, D = \frac{e^2}{8\pi m}\,, \tag{2.47}$$

and we get the spectral function $\overline{\rho(x)}$

$$\frac{m\exp(-m\,|x|)}{4\pi\,|x|} = \int \frac{md^2r}{(2\pi)^2\sqrt{r^2+m^2}}\exp(ir\cdot x)\,, \tag{2.48}$$

$$\begin{aligned}\overline{\rho(x)} &= \frac{m\exp(-m\,|x|)}{4\pi\,|x|}\exp(F(m,x)) \\ &= \frac{m\exp(-(m_0+B)\,|x|)}{4\pi\,|x|}\exp(A)(\mu\,|x|)^{-C|x|+D}\,,\end{aligned} \tag{2.49}$$

where $m = |m_0 + \frac{e^2}{8\pi}(d-2+\gamma)|, B$ denotes the correction of mass, D is a coefficent of the bare Coulomb potential devided by m and $C\,|x|\ln(\mu\,|x|)$ can be understood as the position dependent self-energy as dynamical mass. Here $\exp(-m\,|x|)/4\pi\,|x|$ is a free scalar propagator with physical mass m. In euclidean space we omitt the linear infrared divergent factor A.

2.2 Confining property

Here we mention the confining property of the propagator $S_F(x)$ in position space

$$\begin{aligned}S_F(x) &= \left(\frac{i\gamma\cdot\partial}{m}+1\right)\left[\frac{m\exp(-m\,|x|)}{4\pi\,|x|}(\mu\,|x|)^{D-C|x|)}\right] \\ &= \left(\frac{i\gamma\cdot\partial}{m}+1\right)\overline{\rho(x)}\end{aligned} \tag{2.50}$$

$$D = \frac{e^2}{8\pi m}, C = \frac{e^2}{8\pi}\,. \tag{2.51}$$

Since $(\mu|x|)$ is a dimensionless quantity and taking limit $\mu \to 0$ with finite $|x|$ gives zero except for $|x| = \infty$. In section 3 we take the $\mu \to 0$ limit in momentum space. Thus here we fix $\mu = unit$ of mass as m and see x dependence of the function $(\mu|x|)^{D-C|x|)}$.The $\overline{\rho(x)}$ damps strongly at large x provided

$$\lim_{x\to\infty}(\mu\,|x|)^{-C|x|} = 0\,. \tag{2.52}$$

The profiles of the $\overline{\rho(x)}$ for various values of $D \geq 1$ are shown in figure 1. The effect of $(\mu\,|x|)^{-C|x|}$ in position space is seen to decrease the value of the propagator at low energy and shown in figure 2.

In section 3 and 4 we discuss dynamical mass, the renormalization constant, and bare mass in connection of each terms in F.

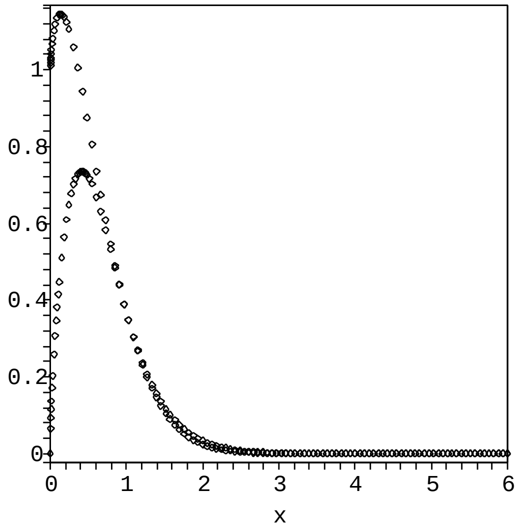

Figure 1: $\overline{\rho(x)}$ for $m = \mu = unit, D = 1, 1.5$.

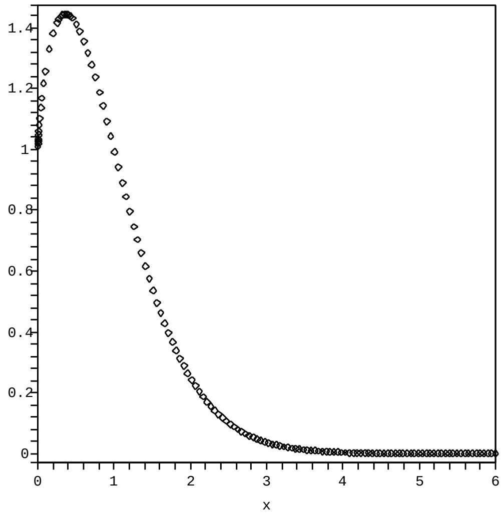

Figure 2: $(\mu|x|)^{-Dm|x|}$ for $m = \mu = unit, D = 1$.

2.3 $O(e^2)$ propagator in momentum space

After angular integration we get the propagator

$$S_F(p) = \left(\frac{\gamma \cdot p}{m} + 1\right) \rho(p)\,, \tag{2.53}$$

$$\rho(p) = \frac{m}{2\pi\sqrt{-p^2}} \int_0^\infty d\,|x| \sin\left(\sqrt{-p^2}\,|x|\right) \exp(A - (m_0 + B)\,|x|)(\mu\,|x|)^{-C|x|+D}\,, \tag{2.54}$$

where

$$A = \frac{e^2}{8\pi\mu}(d-2) + \frac{\gamma e^2}{8\pi m}\,, \quad B = \frac{e^2}{8\pi}(d-2+\gamma)\,, \quad C = \frac{e^2}{8\pi}\,, \quad D = \frac{e^2}{8\pi m}\,. \tag{2.55}$$

If we discuss the euclidean or off-shell propagator we can omitt the linear infrared divergent part in A.In this case m denotes a physical mass

$$m = |m_0 + \frac{e^2}{8\pi}(d-2+\gamma)|\,. \tag{2.56}$$

Here we show the propagator $\rho(p)$ up to $O(e^2)$

$$\begin{aligned}\rho^{(2)}(p) &= \int d^3x \exp(ip\cdot x) \int \frac{md^2r}{r^0} \exp(ir\cdot x) F(x)\\ &= \frac{m}{\sqrt{-p^2}} \int_0^\infty d\,|x| \sin\left(\sqrt{-p^2}\,|x|\right) \exp(-m\,|x|)[1 + A - Cx\ln(\mu\,|x|) + D\ln(\mu\,|x|)]\\ &= \left[\frac{m(1+A)}{m^2+p^2} + m(DI_1 - CI_2)\right],\end{aligned} \tag{2.57}$$

where I_1, I_2 are the following integrals

$$
\begin{aligned}
I_1 &= \int_0^\infty \frac{\sin(\sqrt{-p^2}\,|x|)\exp(-m\,|x|)}{\sqrt{-p^2}} \ln(\mu\,|x|) d\,|x| \\
&= \frac{-\gamma}{m^2+p^2} - \frac{\ln((m^2+p^2)/\mu^2)}{2(m^2+p^2)} - \frac{\ln((m-\sqrt{-p^2})/(m+\sqrt{-p^2}))}{m^2+p^2}, \qquad (2.58)\\
I_2 &= \int_0^\infty \frac{\sin(\sqrt{-p^2}\,|x|)\exp(-m\,|x|)}{\sqrt{-p^2}} |x| \ln(\mu\,|x|) d\,|x| \\
&= \frac{-m}{(m^2+p^2)^2}\left[\ln\left(\frac{(m-\sqrt{-p^2})}{(m+\sqrt{-p^2})}\right) + \ln\left(\frac{(m^2+p^2)}{\mu^2}\right) - 2(1-\gamma)\right]. \qquad (2.59)
\end{aligned}
$$

From these expressions we see that the gauge dependent terms A, D, B, C,are wave function renormalization constant and mass renormalization respectively. In this order the wave function renormalization constant

$$
Z_2^{(2)} = 1 + A - \frac{e^2}{8\pi m}\left(\gamma + \frac{1}{2}\ln\frac{(m^2+p^2)}{\mu^2}\right) + \ln\left(\frac{(m-\sqrt{-p^2})}{(m+\sqrt{-p^2})}\right) \qquad (2.60)
$$

is divergent at $p^2 = -m^2$ and $p^2 = \infty$.

2.4 Mass generation

Since our model QED_3 is super renormalizable mass generation occurs perturbatively. In other words, mass changes as in the case of operator insertion [13]. It is seen by the dimensional analysis of the propagator or self-energy Σ. For instance we take an example for the $O(e^2)$ self-energy

$$
\begin{aligned}
\Sigma^{(2)} &= \gamma\cdot pA(p) + B(p)\,, \\
\Sigma^{(2)} &= e^2 \int \frac{d^3k}{(2\pi)^3}\gamma_\mu S_F(k)\gamma_\nu D_{F0}^{\mu\nu}(p-k)\,, \qquad (2.61)\\
S_F(p) &= \frac{\gamma\cdot pA(p)+B(p)}{A^2(p)p^2+B^2(p)}\,, \qquad (2.62)
\end{aligned}
$$

where$\Sigma^{(2)}$ is given by setting $A=1$, $B=m$ for S_F.By using contour integral we obtain

$$
\begin{aligned}
\frac{1}{2p^2}\operatorname{Tr}\left[(\gamma\cdot p)\Sigma^{(2)}\right] &= A(p) = -\frac{de^2m}{8\pi p^2}\left(1 - \frac{p^2-m^2}{mp}\tan^{-1}\left(\frac{p}{m}\right)\right), \qquad (2.63)\\
\frac{1}{2}\operatorname{tr}(\Sigma^{(2)}) &= B(p) = \frac{(d-2)e^2m}{4\pi p}\tan^{-1}\left(\frac{p}{m}\right). \qquad (2.64)
\end{aligned}
$$

Here we notice that in the limit $B(p)_{p\to 0} = (d-2)e^2/(4\pi)$ which is independent of m.From the above expressions we see that the high-energy behaviour of $A(p)$ and $B(p)$ are proportional to e^2/p in this order with bare vertex. On the other hand, Dyson-Schwinger equation is non-linear

$$
\begin{aligned}
B(p) &= 2e^2\int \frac{d^3k}{(2\pi)^3}\frac{1}{(p-k)^2}\frac{B(k)}{k^2+B(k)^2} \\
&= \frac{e^2}{2\pi^2 p}\int_0^\infty \frac{kdkB(k)}{k^2+B(k)^2}\ln\left(\frac{p+k}{p-k}\right), \qquad (2.65)
\end{aligned}
$$

in the Landau gauge $A = 1$. This yields $B(p) \to m^3/p^2$ as $p \to \infty$.We have the scalar part of the propagator

$$S_E(p) = \frac{1}{4}\,\mathrm{tr}\,S_F(p)_{p\to\infty} \propto \frac{e^4}{p^4}\,, \tag{2.66}$$

which is the correct form of $B(p)$ from dimensional analysis [13]. Here we return to our approximation. The propagator in momentum space is

$$\begin{aligned} S_F(p) &= -\int d\omega^2 \frac{\gamma\cdot p\rho_1(\omega^2)+\rho_2(\omega^2)}{p^2+\omega^2+i\epsilon} = \left(\frac{\gamma\cdot p}{m}+1\right)\rho(p)\,, && (2.67)\\ \rho(p) &= \int d^3x\, \exp(-ip\cdot x)\overline{\rho(x)}\,, && (2.68)\\ \overline{\rho(x)} &= \frac{m\exp(-m\,|x|)}{4\pi\,|x|}(\mu\,|x|)^{-C|x|+D}\,. && (2.69) \end{aligned}$$

From the above equations the conditions for mass generation is easily seen that

$$\begin{aligned} &0 \le \overline{\rho(x)} \le \mathrm{Max}\,, \qquad \text{for} \qquad 1 \le D\,, && (2.70)\\ \rho(p=\infty) &= \lim_{p\to\infty}\frac{1}{\sqrt{-p^2}}\int_0^\infty x^2\frac{\sin(\sqrt{-p^2}\,|x|)}{|x|}\overline{\rho(x)} = \lim_{p\to\infty}\frac{1}{\sqrt{-p^2}}\pi(x^2\overline{\rho(x)})_{x=0} = 0\,, && (2.71)\\ \rho(p=0) &= \int d^3x\overline{\rho(x)} = \mathrm{finite}(\simeq 0.53\ \mathrm{for} D=1, m=\mu=1)\,, && (2.72) \end{aligned}$$

which are a prioli assumed to be satisfied in the numerical analysis of Dyson-Schwinger equation.

3. Analysis in momentum space

To search the infrared behaviour we expand the propagator in the coupling constant e^2 and obtained the Fourier transform of $\overline{\rho(x)}$ [4]. In that case it is not enough to see the structure of infrared behaviour which can be compared to the well-known four dimensional QED. Instead we make Laplace transformation of $(\mu|x|)^{-C|x|}$,which leads the general spectral representation of the propagator in momentum space. After that we show the roles of Coulomb energy and position dependent mass. The former determines the dimension of the propagator and the latter acts to change mass. Let us begin to study the effect of position dependent mass(Self-energy), Coulomb energy in momentum space

$$\begin{aligned} \exp(-\,|x|\,M(x)) &= \exp\left(-\frac{e^2}{8\pi}\,|x|\ln(\mu\,|x|)\right)\,, && (3.1)\\ \exp\left(\frac{\text{Coulomb energy}}{m}\right) &= \exp\left(\frac{e^2}{8\pi m}\ln(\mu\,|x|)\right)\,. && (3.2) \end{aligned}$$

Similar discussion was given to study the effects of self-energy and bare potential in the stability of massless e^+e^- composite in the lattice simulation [17]. The position space free propagator

$$S_F(x,m_0) = -(i\gamma\cdot\partial+m_0)\frac{\exp(-m_0\,|x|)}{4\pi\,|x|} \tag{3.3}$$

is modified by these two terms which are related to dynamical mass and wave function renormalization. To see this let us think about position space propagator

$$\overline{\rho(x)} = \frac{m\exp(-m\,|x|)}{4\pi\,|x|}(\mu\,|x|)^{-\frac{e^2}{8\pi}|x|}(\mu\,|x|)^{\frac{e^2}{8\pi m}}\,. \tag{3.4}$$

It is easy to see that the probability of particles which are separated with each other in the long distance is supressed by the factor $(\mu|x|)^{-C|x|}$,and the Coulomb energy modifies the short distance behaviour from the bare $1/|x|$ to $1/|x|^{1-D}$. The effect of Coulomb energy for the infrared behaviour of the free particle with mass m can be seen by its fourier transform [4]

$$\begin{aligned}\int d^3x\exp(-ip\cdot x)\frac{\exp(-m\,|x|)}{4\pi\,|x|}(\mu\,|x|)^D &= \mu^D\frac{\Gamma(D+1)\sin((D+1)\arctan(\sqrt{-p^2}/m))}{\sqrt{-p^2}(p^2+m^2)^{(1+D)/2}}\\ &\sim \mu^D(\sqrt{-p^2}-m)^{-1-D}\ \text{near}\ p^2=-m^2\,.\end{aligned} \tag{3.5}$$

Above formula shows the structure in momentum space is modified for both infrared and ultraviolet regions. Usually constant D represents the coefficent of the leading infrared divergence for fixed mass in four dimension. Therefore Coulomb energy has the same effects in three dimension as in four dimension but change the ultraviolet behaviour since the coupling constant e^2 is not renormalized. Now we consider the role of $M(x)$ as the dynamical mass at low momentum. First we define Fourier transform of the $\overline{\rho(x)}$;

$$\rho(p) = \int\exp(-ip\cdot x)\overline{\rho(x)}d^3x\,. \tag{3.6}$$

Momentum dependence of $\rho(p)$ for various values of D is shown numerically in figure 3. We notice that

$$D=0\to\rho(p)=\frac{m}{p^2+m^2}\,, \tag{3.7}$$

$$D=1\to\rho(p)=\frac{2m^2\mu}{(p^2+m^2)^2}\times\log\ \text{correction}\,, \tag{3.8}$$

which we can see from (2.69), (2.57)–(2.59). If we include $(\mu|x|)^{-C|x|}$ term it is easy to see that the value of the proagator $\rho(p=0)$ decreases which is shown numerically in figure 3. In the previous paper we expand the propagator $\rho(p)$ in powers of e^2 and see the logarithmic infrared divergence at $p^2=m^2$. In that case it is not clear the effect of position dependent mass. If we use Laplace transformation it is easily seen that $(\mu|x|)^{-C|x|}$ acts as mass changing operator $m\to m-s$

$$\begin{aligned}F(s) &= \int_0^\infty\exp(-s\,|x|)(m\,|x|)^{-C|x|}d\,|x|\,,\\ \exp(-m^*\,|x|)(m\,|x|)^{-C|x|} &= \int_0^\infty\exp(-(m^*-s)\,|x|)F(s)ds,\end{aligned} \tag{3.9}$$

where we introduced

$$m^* = m + C\ln\left(\frac{\mu}{m}\right), \tag{3.10}$$

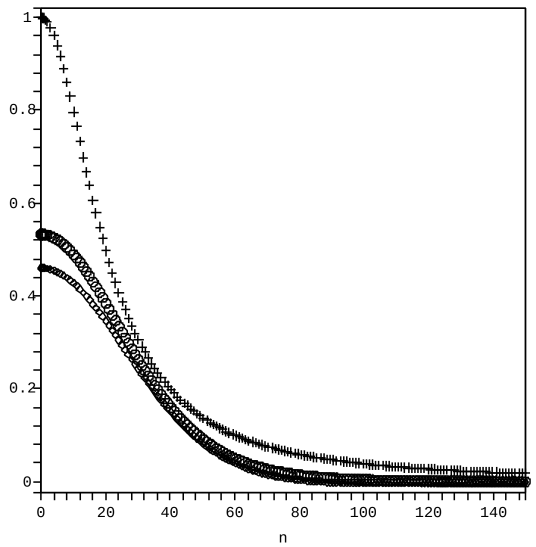

Figure 3: $\rho(p)$ for $m = \mu = unit$, $D = 0$(upper), 1(middle), 1.5(upper), $p = n/20$.

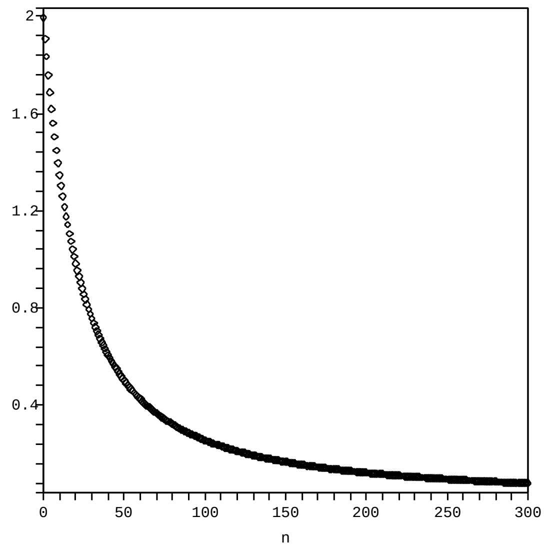

Figure 4: Laplace transform $F(s)$ for $(m|x|)^{-Dm|x|}$, $m = 1$, $D = 1$, $s = n/20$.

to separate the cut-off μ dependence from $F(s)$ in the following way

$$\exp(-m\,|x)|\,(\mu\,|x|^{-C|x|} = \exp(-m^*\,|x|)(m\,|x|)^{-C|x|}. \tag{3.11}$$

The Laplace transfom $F(s)$ is shown in figure 4.

We have the complete the expression of the propagator

$$\overline{\rho(x)} = \frac{m\mu^D \exp(-m^*\,|x|)}{4\pi\,|x|^{1-D}} \int_0^\infty \exp(s\,|x|)F(s)ds\,, \tag{3.12}$$

$$\rho(p) = m\mu^D \int_0^\infty \frac{\Gamma(D+1)\sin((D+1)\arctan(\sqrt{-p^2}/(m^*-s)))F(s)ds}{\sqrt{-p^2}\sqrt{(p^2+(s-m^*)^2)^{1+D}}}\,, \tag{3.13}$$

$$\rho(p=0) = \int_0^\infty \frac{2(s-m^*)F(s)ds}{(s-m^*)^4} = \frac{1}{2\pi i}\int_{s-i\infty}^{s+i\infty} \frac{2F(s)ds}{(s-m^*)^3} \simeq 0.53 \text{ for } m = \mu = 1, D = 1\,. \tag{3.14}$$

Here we study the cut off $\mu \to 0$ limit. We see that m^* diverges as $\ln(\mu/m)$ in the limit $\mu \to 0$. In this case we can neglect the s dependence of the denominator of $\rho(p)$ and using

$$0 \le \frac{s-m^*}{((p^2+(s-m^*)^2)^{\frac{1+D}{2}}} \le \frac{s-m^*}{(p^2+m^{*2})^{\frac{!+D}{2}}}\,,$$
$$\int_0^\infty F(s)ds = 1\,, \qquad \int_0^\infty sF(s)ds = \text{finite}\,, \tag{3.15}$$

we get the infrared behaviour

$$\rho(p) \approx \frac{m^{1+D}\epsilon^D\Gamma(1+D)\sin((D+1)\arctan(\sqrt{-p^2}/(s-m^*))}{\sqrt{-p^2}(p^2+(s-m^*)^2)^{\frac{1+D}{2}}} \to 0\,, \quad \epsilon = \frac{\mu}{m} \to 0\,, \tag{3.16}$$

and for $D = 1$

$$\rho(p) \approx \frac{m^2\epsilon(1+\ln(\epsilon))}{(p^2+m^2(1+\ln(\epsilon))^2)^2} \to 0\,. \tag{3.17}$$

In Minkowski space we have by replacement $p^2 \to -p^2$ with infrared factor $\exp(A)$,

$$A = \frac{e^2}{8\pi\mu}(d-2) + \frac{e^2}{8\pi m}\,. \tag{3.18}$$

Since A is highly gauge dependent we do not have a definite answer in the limit $\mu \to 0$ except for the Yennie gauge $d = 2$. The infrared behaviour near $p^2 = m^2$ becomes

$$\rho(p) \approx \frac{\Gamma(D+1)\epsilon^D \sin((D+1)\arctan(p/m(1+D\ln(\epsilon)))}{(-p^2+m^2(1+D\ln(\epsilon))^2)^{1+D/2}} \to 0\,,$$

and for $D = 1$

$$\rho(p) \approx \frac{2m^3\epsilon(1+\ln(\epsilon))}{(-p^2+m^2(1+\ln(\epsilon))^2)^2} \to 0\,, \tag{3.19}$$

this leads to

$$\rho(p) \approx \frac{2\epsilon}{m(\ln(\epsilon))^4} = \frac{2\sqrt{1-p^2/m^2}}{m(\ln(\sqrt{1-p^2/m^2}))^4}\,, \quad \text{for} \quad D=1, d=2\,, \tag{3.20}$$

near $p^2 = m^2$.In Euclidean space it is natural that in the long-distance the propagator vanishes in the limit $\mu \to 0$.In Minkowski space we find a vanishiment of the propagator as $\sqrt{1-p^2/m^2}(\ln(1-p^2/m^2))^{-4}$ near $p^2 = m^2$ for the Yennie gauge at $D = 1$, which is a consequence of confinement picture in our model. In comparison between our model and Dyson-Schwinger equation, $D = 1$ is prefered for dynamical mass generation. It has been discussed that the propagator will have a branch point on the real p^2 axis associated with dynamical mass [18, 19]. Therefore we can say that the radiatively corrected fermion has not a simple structure as in QED_4,in QED_3 it has a superposition of different mass with cut and linear divergence associated with massless photon in the ordinary phase or it is expressed as superposition of dipoles with linear divergence in the condensed phase for finite cut-off μ, which will be shown in the next section. There is a possibility to remove infrared cut-off by including photon mass as Chern-Simon term or vacuum polarization of photon [5, 8, 9, 11, 12].

4. Bare mass and vacuum expectation value $\langle\overline{\psi}\psi\rangle$

In this section we examine the renormalization constant and bare mass and study the condition of vanishing bare mass based on spectral representation. The equation for the renormalization constant in terms of the spectral functions read

$$\lim_{p\to\infty} \frac{Z_2^{-1}(\gamma\cdot p + m_0)}{p^2 - m_0^2 + i\epsilon} = \lim_{p\to\infty} \int \frac{\gamma\cdot p\rho_1(\omega^2) + \rho_2(\omega^2) d\omega^2}{p^2 - \omega^2 + i\epsilon}\,. \tag{4.1}$$

Instead we determine them directly by taking the high energy limit of $S_F(p)$

$$m_0 Z_2^{-1} = m\int \rho_2(\omega^2)d\omega^2 = \frac{1}{4}\lim_{p\to\infty} \mathrm{tr}\left[p^2 S_F(p)\right] \tag{4.2}$$

$$Z_2^{-1} = \int \rho_1(\omega^2)d\omega^2 = \frac{1}{4}\lim_{p\to\infty} \mathrm{tr}\left[\gamma\cdot p S_F(p)\right]. \tag{4.3}$$

In (3.13) the function $\sin((D+1)\arctan(\sqrt{-p^2}/(m^*-s))$ is expanded for large $(-\,p^2)$,

$$\sin((D+1)\arctan(\frac{\sqrt{-p^2}}{m^*-s}) = \sin(\frac{(D+1)}{2}\pi) - \cos(\frac{(D+1)}{2}\pi)\frac{(D+1)(m^*-s)}{\sqrt{-p^2}} + O(\frac{1}{-p^2})\,. \tag{4.4}$$

From this we obtain for $D<1$

$$m_0 Z_2^{-1} = m\mu^D\Gamma(D+1)\sin\left(\frac{(D+1)\pi}{2}\right)\lim_{p^2\to\infty}\sqrt{-p^2}^{\,-D} = \begin{bmatrix} 0 & (0<D) \\ m & (0=D) \end{bmatrix}, \tag{4.5}$$

$$Z_2^{-1} = \mu^D\Gamma(D+1)\sin\left(\frac{(D+1)\pi}{2}\right)\lim_{p^2\to\infty}\sqrt{-p^2}^{\,-D} = \begin{bmatrix} 0 & (0<D) \\ 1 & (0=D) \end{bmatrix}. \tag{4.6}$$

For $D=1$ case first term in the $1/\sqrt{-p^2}$ expansion vanishes and

$$m_0 Z_2^{-1} = \lim_{p\to\infty}\frac{2m\mu}{-p^2}\int(m^*-s)F(s)ds = 0\,, \tag{4.7}$$

$$Z_2^{-1} = \lim_{p\to\infty}\frac{2\mu}{-p^2}\int(m^*-s)F(s)ds = 0\,. \tag{4.8}$$

This means that propagator in the high energy limit has no part which is proportional to the free one and it shows confinement. Usually mass is a parameter which appears in the Lagrangean. For example chiral symmetry is defined for the bare quantity. In ref. [14] the relation between bare mass and renormalized mass of the fermion propagator in QED is discussed based on renormalization group equation with the assumption of ultraviolet stagnant point and shown that the bare mass vanishes in the high energy limit even if we start from the finite bare mass in the theory. It suggests that symmetry properties can be discussed in terms of renormalized quantities. In QCD bare mass vanishes in the short distance by asymptotic freedom. And the dynamical mass vanishes too [13]. Since our model is super-reomalizable bare mass m_0 does not vanish. In our apprximation this problem is understood that at short distance propagator in position space tends to

$$\overline{\rho(x)} = \frac{m\exp(-m_0\,|x|)}{4\pi\,|x|}\exp(-B\,|x| + D\ln(\mu\,|x|) - C\,|x|\ln(\mu\,|x|))_{x\to 0} \tag{4.9}$$

$$\to |x|^{D-1}(\mu\,|x|)^{-C|x|}, \tag{4.10}$$

where we have $\overline{\rho(0)} = \mathit{finite}$ at $D=1$ case which is independent of the bare mass m_0. Thus we have a same effect as vanishing bare mass in four dimensional model. Of course we have a dynamical mass generation which is $m = |\frac{e^2}{8\pi}(d-2+\gamma)| + M(x)$ for $m_0 = 0$ in our approximation.There is a chiral symmetry at short distance where the bare or dynamical mass vanishes in momentum space but its breaking must be discussed in terms of the values of the order parameter. Therefore it is interesting to study the possibility of pair condensation in our approximation. The vacuum expectation value of pair condensate is

JHEP09(2004)048

evaluated

$$
\begin{aligned}
\langle\overline{\psi}\psi\rangle &= -\operatorname{tr} S_F(x) = -2m\mu^D \lim_{x\to 0_+} \frac{\exp(-m\,|x|)}{|x|}(\mu\,|x|)^{-C|x|+D} \\
&= \begin{pmatrix} 0 & (D>1) \\ \text{finite} & (D=1) \\ \infty & (D<1) \end{pmatrix},
\end{aligned} \tag{4.11}
$$

for finite cut-off μ.For $D<1$ case the $\overline{\rho(x)}$ is divergent at $x=0$ and it looks like a spike, for $D=1, \overline{\rho(0)} = finite$ and it looks like a wave packet at finite range where long range correlation appears, and for $D>1, \overline{\rho(0)} = 0$ but it does not damp fast for large x. In the weak coupling limit we obtain $Z_2 = 1$, $m_0 = m$ and $\langle\overline{\psi}\psi\rangle = \infty$. If we introduce chiral symmetry as a global U($2n$), it breaks dynamically into SU(n) $\times$ SU(n) $\times$ U(1) $\times$ U(1) as in QCD [13, 16] for $D=1$ for finite infrared cut-off. Our model may be applicable to relativistic model of super fluidity in three dimension. Usually we do not find the critical coupling $D=1$ in the analysis of the Dyson-Schwinger equation in momentum space where only continuum contributions are taken into account and we do not define physical mass.

5. Summary

Infrared behaviour of the propagator has been examined in the Bolch-Nordsieck-like approximation to QED$_3$. Using LSZ reduction formula for the one photon matrix element we find two kinds of logarithmic infrared divergent terms. These are position dependent mass and Coulomb energy both of which are gauge independent in our approximation. Confining property is shown by position dependent mass at large distance where propagator vanishes faster than the sloution of Dyson-Schwinger equation converted into position space with some approximation. Renormalization constant Z_2^{-1} vanishes for arbitrary coupling D in our approximation, which implies confinement. Momentum space propagator is expressed as dispersion integral and it shows us that the effects of dynamical mass can be written as a superposition of the propagator with different masses. In our approximation there seems to be a critical coupling constant for vacuum expectation value $\langle\overline{\psi}\psi\rangle$ which is independent of the bare mass. Our model shows that the confining property of charged fermion and dynamical mass generation are realized in QED_3 (at critical coupling constant $D=1$) as in QCD in four dimension with finite infrared cut-off which may be supplied by some dynamical origin of the photon mass beyond the quenched approximation .

Acknowledgments

The author would like thank Prof. Roman Jackiw at MIT and Prof. Robert Delbourgo at University of Tasmania for their hospitalities and communications on our subjects on infrared problems in three dimensional gauge theory. He also thanks to Dr. Suzuki at Hokkaido University for his interest for the infrared limit.

A. Gauge dependence

In ref. [4, 7, 8, 9, 14] gauge dependece of the propagator has been discussed. In order to understand the gauge dependence of the propagator away from the threshold, we will once examine the gauge covariance relation. Writing $D_{\mu\nu}(k) = D^{(0)}_{\mu\nu}(k) + k_\mu k_\nu M(k)$,we see that the fermion proagator varies with the gauge according to [6],

$$S_F(x, K) = \exp\left(-iK\sqrt{x^2}\right) S_F(x, 0)\,, \tag{A.1}$$

$$M(x) = ie^2 d \int \frac{d^3k}{(2\pi)^3}\frac{1}{k^4}\left(\exp(-ik\cdot x) - 1\right) = -iK\sqrt{x^2}\,, \tag{A.2}$$

where $K = e^2d/8\pi$. Thus the gauge transformation property is violated by linear infrared divergences near the threshold. Here we adopt the subtracion of the point $k = 0$ in the integral. Free propagator with mass ω equals to

$$S^{(0)}_F(x) = -(i\gamma\cdot\partial + \omega)\frac{\exp(-\omega\sqrt{-x^2})}{4\pi\sqrt{-x^2}}\,, \tag{A.3}$$

and its Fourier transform is

$$S^{(0)}_F(p) = \int d^3x \exp(ip\cdot x) S^{(0)}_F(x) = \frac{\gamma\cdot p + \omega}{p^2 - \omega^2}\,. \tag{A.4}$$

In our approximation only mass terms changes under the gauge transformation and remaining terms in F are gauge invariant. In general gauge invariant part is approximation dependent.

Finally we show the general spin dependent spectral function in the $O(e^2)$ for d gauge by evaluating

$$F_v = \frac{1}{4r^2}\mathrm{tr}(\gamma\cdot rF)\,, \qquad F_s = \frac{1}{4}\mathrm{tr}(F)\,,$$

and we have

$$\begin{aligned} F &= \left(\frac{\gamma\cdot r}{m} + 1\right)\left[\frac{e^2}{8\pi}(-(d-2+\gamma)\,|x| - \frac{e^2}{8\pi}\,|x|\ln(\mu\,|x|) + \frac{e^2}{8\pi m}(\ln(\mu\,|x|) + \gamma)\right] - \\ &\quad - \frac{\gamma\cdot r}{m}\frac{e^2(d-1)}{8\pi m^2\,|x|}. \end{aligned} \tag{A.5}$$

The first term is the same as the previous one. The second term is not significant in the infrared. This term is also obtained by gauge transformation from the Landau gauge where $\rho_1 = \rho_2$ is assumed [11, 12]. Since

$$P = \frac{\gamma\cdot r + m}{2m}$$

JHEP09(2004)048

is a projection operator it is easy to show the following. In Euclidean space if we exponentiate we have

$$\begin{aligned}
S_F(x) &= -\int \frac{mrdr}{\sqrt{r^2+m^2}} \exp(ir\cdot x)\left(1+\frac{r\cdot\gamma}{m}\right)\exp\left(\left(\frac{r\cdot\gamma}{m}+1\right)F\right) \\
&= -\int \frac{mrdr}{\sqrt{r^2+m^2}} \exp(ir\cdot x)\left(1+\frac{r\cdot\gamma}{m}\right)\exp\left(\frac{F}{2}\right)\left(\cosh\left(\frac{F}{2}\right)+\frac{r\cdot\gamma}{m}\sinh\left(\frac{F}{2}\right)\right) \\
&= -\int \frac{mrdr}{\sqrt{r^2+m^2}} \exp(ir\cdot x)\left(1+\frac{r\cdot\gamma}{m}\right)\exp(F)\,, \qquad \text{(A.6)}
\end{aligned}$$

$$S_F(p) = \int d^3x \exp(-ip\cdot x) S_F(x)\,. \qquad \text{(A.7)}$$

References

[1] R. Jackiw, L. Soloviev, *Low-energy-theorem approach to single-particle singularities in the presence of massless bosons*, *Phys. Rev.* **173** (1968) 1485.

[2] F.E. Low, *Bremsstrahlung of very low-energy quanta in elementary particle collisions Phys. Rev.* **110** (1958) 974;
M. Gell-Mann, M.L. Goldberger. *Scattering of low-energy photons by particles of spin* 1/2, *Phys. Rev.* **96** (1954) 1433;
S. Weinberg, *Infrared photons and gravitons*, *Phys. Rev.* **140** (1965) 516;
R. Jackiw, *Low-energy theorems for massless bosons: photons and gravitons*, *Phys. Rev.* **168** (1968) 1623;
L.D. Landau and E.M. Lifzhits, *Quantum electrodynamics*, Pergamon Press, Oxford, 1982;

[3] C. Itzykson, J.B. Zuber, *Quantum field theory*, McGRAW-HILL.

[4] Y. Hoshino, *Low-energy theorem approach to one-particle singularity in* QED_{2+1}, *J. High Energy Phys.* **05** (2003) 075 [hep-th/0304021].

[5] S. Deser, R, Jackiw, S. Templton, *Topologically massive gauge theories*, *Ann. Phys. (NY)* **140** (1982) 372.

[6] L.D. Landau, Khalatonikov, Zh. Eksp. Theor. Fiz. 29 (1955) 89[Sov. Phys. JETP2(1956)69];
B. Zumino, *Gauge properties of propagators in quantum electrodynamics*, *J. Math. Phys.* **1** (1960) 1.

[7] C.J. Burden, J. Praschifka and C.D. Roberts, *Photon polarization tensor in three-dimensional quantum electrodynamics*, *Phys. Rev.* **D 46** (1992) 2695.

[8] C.J. Burden and C.D. Roberts, *Gauge covariance and the fermion-photon vertex in three-dimensional and four-dimensional, massless quantum electrodynamics*, *Phys. Rev.* **D 47** (1993) 5581 [hep-th/9303098].

[9] A. Bashir and A. Raya, *Constructing the fermion boson vertex in* QED_3, *Phys. Rev.* **D 64** (2001) 105001 [hep-ph/0103306].

[10] T. Appelquist and R.D. Pisarski, *High-temperature Yang-Mills theories and three-dimensional quantum chromodynamics*, *Phys. Rev.* **D 23** (1981) 2305;
T. Appelquist, U. Heinz, *Phys. Rev.* **D 24** (1981) 2169.

[11] A.B. Waites, R. Delbourgo, *Nonperturbative behavior in three-dimensional QED*, *Int. J. Mod. Phys.* **A 7** (1992) 6857.

[12] E. Abdalla, R. Banerjee, C. Molina, Eur. Phys. J. C. 17 (2000) 467.

[13] H.D. Politzer, *Effective quark masses in the chiral limit*, *Nucl. Phys.* **B 117** (1976) 397.

[14] K. Nishijima, *Condition for the vanishing bare fermion mass*, *Prog. Theor. Phys.* **81** (1989) 878; *Fermion mass generation and the Schwinger-Dyson equation*, *Prog. Theor. Phys.* **83** (1990) 1200.

[15] L.S. Brown, *Quantum field theory*, Cambridege University Press, 1992;
N.N. Bogoliubov, D.V. Shirkov, *Introduction to the theory of quantized fields*, Wiley-Interscience, section 32;
L.D. Landau and L. Lifzhits, *Quantum electrodynamics*, Pergamon Press, Oxford, 1982.

[16] T. Appelquist, D. Nash, L.C.R. Wijewardhana, *Phys. Rev. Lett.* **60** (1988) 2575;
Y. Hoshino and T. Matsuyama, *Dynamical parity violation in QED in three-dimensions with a two component massless fermion*, *Phys. Lett.* **B 222** (1989) 493.

[17] E. Dagotto, J.B. Kogut and A. Kocic, *A computer simulation of chiral symmetry breaking in (2+1)-dimensional QED with N flavors*, *Phys. Rev. Lett.* **62** (1989) 1083.

[18] D. Atkinson and D.W.E. Blatt, *Determination of the singularities of the electron propagator*, *Nucl. Phys.* **B 151** (1979) 342.

[19] M. Koopman, *Dynamical mass generation in* QED_3, Ph.D thesis, Groningen University, 1990, chapter 4;
P. Maris, *Confinement and complex singularities in QED in three-dimensions*, *Phys. Rev.* **D 52** (1995) 6087 [hep-ph/9508323];
Y. Hoshino, *Complex singularity of the fermion propagator in quenched* QED_3, *Nuovo Cim.* **A 112** (1999) 335.